Note: Remarks on Modeling the Formation of Uranus and Neptune

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ABSTRACT

We have studied two scenarios for the *in situ* formation of Uranus and Neptune from a hundred or so sub-Earth-sized planetary embryos initially on low-inclination, nearly-circular orbits beyond Saturn. We find that giant planets do not form during integrations of such systems. Almost no accretion occurs at all because the embryos are dynamically excited by each other and the gravitational effects of Jupiter and Saturn on a timescale that is short compared to the collision timescale. This produces large eccentricities and inclinations which significantly decrease the the collisional cross-section of the embryos because it decreases the effects of gravitational focusing. As a result, giant planets do not grow. These simulations show that the *standard* model for the formation of the Uranus and Neptune is most likely not correct.

Subject headings: solar system: formation

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1. Introduction

In the last decade, it has become apparent that the processes that formed Uranus and Neptune are fundamentally different from those that formed the other giant planets (see Lissauer et al. 1995). The absence of massive hydrogen atmospheres suggests that the final stages of accretion of Uranus and Neptune post-dates the dissipation of the solar nebula and consequently, the formation of Jupiter and Saturn (Lissauer et al. 1995). Thus, one is faced with the problem of accretion in a gas-free environment. In this way, the formation of Uranus and Neptune is somewhat reminiscent of the formation of the terrestrial planets. However, unlike the terrestrial planets, Uranus and Neptune formed in a region of the solar system where the escape velocity of the planetary embryos is comparable to their orbital velocity.

It has often been assumed that the formation of Uranus and Neptune can be divided into three broad, but distinct, stages: *i*) the formation of ~ 10 km sized objects (Weidenschilling & Cuzzi 1993; Ward 2000), *ii*) the growth of massive planetary embryos through runaway growth until the embryos grow large enough to perturb one another into crossing orbits (Wetherill & Stewart 1993), and *iii*) the merger of these planetary embryos into Uranus and Neptune size planets (Ip 1989, hereafter I89).

There has been an ongoing controversy concerning the last stage of Uranus and Neptune formation (*iii* above). I89 followed the evolution of one hundred Mars- to Earth-sized planetesimals beyond the orbit of Saturn. Ip's calculations are based on a Monte Carlo method where successive close encounters and collisions are selected probabilistically based on Öpik's formula for two-body interactions (Öpik 1951). He reports that Uranus and Neptune-sized planets in low-eccentricity orbits are formed in a few $\times 10^8$ years. These planets were located roughly at the present locations of Uranus and Neptune in the real solar system. Fernández & Ip (1996) report similar results starting from a swarm of 750 planetesimals with masses 1/7–1/8 as large as before.

However, using similar techniques Lissauer et al. (1995) were unable to reproduce Ip's results. Indeed, they found that almost no accretion occurred during the age of the solar system. In 1998, we attempted to reproduce Ip's calculations using direct orbital integrations, and also found that very little accretion occurred in our 10^8 year simulation. The results of this integration were never published[1] and are discussed in more detail below.

Brunini & Fernández (1999, hereafter BF99) recently reported on a series of direct

[1] The results were presented at the 1998 LPSC meeting. However, no details of the simulation were presented in the published LPSC abstract (see Stewart & Levison 1998).

numerical integrations of the late stages of Uranus-Neptune formation. They followed the behavior of several hundred objects each of roughly 1/4 $M_\oplus$. These objects were spread between 13 and 32 AU with eccentricities up to 0.1. These authors found that objects of similar mass to Uranus and Neptune form in a few tens of millions of years.

We found BF99's result surprising given our results mentioned above. Thus we contacted Adrian Brunini and suggested that a comparison be done between our two codes. We also contacted John Chambers and Shigeru Ida and requested that they participate in this comparison. Brunini kindly supplied us with his initial conditions to one of his runs. Thus, we had the same set of initial conditions being integrated using 4 independent codes. The results of our group and those of Chambers and Ida agree quite well, while Brunini & Fernández reported merger rates nearly an order of magnitude larger than the other three groups. Brunini (2000; pers. comm.) has since discovered a problem with his simulations and now acknowledges that Uranus and Neptune should not have formed in his simulations.

So, there is now apparently consensus that Uranus and Neptune will not form directly from a set of hundreds of Mars- to Earth-sized embryos evolving solely under the gravitational effects of the Sun, Jupiter, Saturn and each-other. Despite this agreement, a search of the refereed literature would lead one to a very different conclusion. I89, Fernández & Ip (1996), and BF99 all appear in a refereed journals and these integrations are the only topic discussed. In Lissauer et al. (1995), this topic is only a very small part of an extensive discussion of all aspects of Uranus-Neptune formation and has been overlooked in subsequent manuscripts. A detailed description of our work on this topic has not yet been published.

Thus, we are compelled to discuss our simulations in this *Note*. Below, we present the results of two simulations; both of which attempt to reproduce previously published works. In §2, we present a direct N-body integration based on the Monte Carlo simulations in I89, while in §3 we present a simulation using BF99's initial conditions. We conclude in §4.

2. Comparison with I89's Monte Carlo simulations

In the simulation reported in this section we followed the first set of initial conditions described in I89. The simulations initially consist of Jupiter, Saturn, and 100 planetary embryos. The initial semi-major axis distribution of the embryos followed a Gaussian law with a mean of $a_0 = 15\,\mathrm{AU}$ and a dispersion of $\Delta a_0 = 5\,\mathrm{AU}$. The initial eccentricities and inclinations of the embryos were set to 0.05. Their initial longitude of perihelion, the longitude of the ascending node and the mean anomaly were chosen randomly from a uniform distribution between 0 and 2π. Each embryo had an initial mass of $0.63\,M_\oplus$ making the total mass of embryos equal to twice the total mass of Uranus and Neptune. The embryos had an initial density of 1 $\mathrm{g/cm^3}$.

We have integrated the orbits of these embryos using a full N-body, symplectic algorithm known as SyMBA (Duncan et al. 1998). This code has the speed of the sophisticated, highly efficient computer algorithms developed by Wisdom & Holman (1991), but in addition it can accurately handle close encounters between objects. Our technique is based on an algorithm that is similar to the mapping of Wisdom & Holman, but it integrates through encounters by employing a variant of the multiple step-size techniques developed by Skeel & Biesiadecki (1994). In addition, it has been modified to handle small perihelion distances using techniques developed by Levison & Duncan (2000). The integration was done using a timestep of 0.25 years and lasted a total of 10^8 years[2]. If during the integration, two objects came within the sum of their physical radii of one another, they were assumed to merge into a single object in which mass and linear momentum was conserved. Objects were removed from the system if they reached 1000 AU from the Sun and were unbound from the system.

Starting with these initial conditions, I89 reports a significant amount of growth occurs in the first 2×10^7 years. For example, in the simulations illustrated in I89's Figure 2, after 2×10^7 years there are three large objects beyond the orbit of Saturn. These objects have masses of $3\,M_\oplus$, $5\,M_\oplus$, and $23\,M_\oplus$. At 4 Gyr, I89 reports two massive objects beyond Saturn with masses of $4.5\,M_\oplus$ and $25\,M_\oplus$. These objects have similar masses and semi-major axes to Uranus and Neptune. Thus, I89 concludes that Uranus and Neptune formed in a similar fashion in this way.

The results we obtained are very different from those of I89. After 10^8 years, there had not been a single impact between the embryos. Indeed, only one collision had occurred which was between Jupiter and an embryo. We can shed some light on the reason for

[2]We have run shorter experiments with smaller timesteps and find that our results are not sensitive to the timestep.

the low number of collisions in our simulation by considering a simple particle-in-a-box approximation. Using the initial orbital elements in the simulation, we find that the impact rate on each individual embryo is about one collision every 4×10^7 years — too long to expect any significant accretion in the time it takes Uranus and Neptune to form in I89's simulations.

The system in our simulation behaves in the following way. Initially the system dynamically heats up due to the mutual gravitational interactions of the embryos and the effects of Jupiter and Saturn. Recall that at the beginning of the simulation, the embryos all had an eccentricity of 0.05. By 10^4 years the root-mean-square (RMS) eccentricity is 0.11. The RMS eccentricity reaches a maximum of 0.35 after $\sim 4 \times 10^5$. At this time the simple particle-in-a-box approximation gives the impact rate on each individual embryo of about one collision every 10^8 years.

The subsequent behavior is best illustrated by Figure 1, which shows the state of the system at $t = 5 \times 10^5$ years. In the figure we plot the semi-major axes of a particle on the ordinate and its radial distance on the abscissa. The horizontal 'errorbars' on the plot show the range of radial extent of the heliocentric distances due to a particle's eccentricity. The dotted line shows the location of Saturn. As the eccentricity of the embryos grow, eventually they evolve onto orbits that cross the orbit of Saturn. After an embryo reaches this point, its evolution is then dominated by Saturn (and to a lesser extent Jupiter). Typically these particles evolve on orbits that have fixed perihelion distances and rapidly changing semi-major axes. Saturn pushes the particles to large semi-major axes; resulting in ejection from the Solar System or placement in the Oort cloud. All objects with $a \gtrsim 30\,\mathrm{AU}$ are in this phase of their evolution.

Thus, the region just beyond Saturn is slowly depleted of objects and nothing grows there. This is illustrated in Figure 2, which shows the total amount of mass contained in the embryos as a function of time in our simulations. By the end of the simulation, roughly half of the embryo mass had been ejected from the system and there is too little left to form Uranus and Neptune.

3. Comparison with BF99's N-Body simulations

In the simulation described in this section, we make use of a set of initial conditions similar to those in BF99's Run 26. The simulation integrated the orbits of Jupiter, Saturn, and 250 planetary embryos using the methods described above. The initial semi-major axes of the embryos ranged from 12 to 35 AU and followed a surface density $\propto r^{-1.5}$ after Weidenschilling (1977). Their inclinations and eccentricities were chosen randomly from a uniform distribution with a maximum value of 0.01. Their initial longitude of perihelion, the longitude of the ascending node and the mean anomaly were chosen randomly from a uniform distribution between 0 and 2π.

The embryos all had a mass of $0.27\,M_\oplus$ and the sum total of their mass was $67\,M_\oplus$. The embryos had a density of 0.46 g/cm^3. This is roughly 3 times smaller than that of Uranus and Neptune. Thus, the physical cross section of the embryos is about twice as large as they would be if a more reasonable value of the density were used. This in turn, artificially enhances the collision rate of this simulation. This fact should be taken into account when the results presented below are considered.

Compared to the simulations in §2, this run begins with a larger number of smaller embryos which are distributed further from Saturn and have smaller initial orbital eccentricities. Note that all these changes should favor accretion. Although we initially planned to integrate the system for much longer, we stopped this simulation at 8×10^6 years when it became clear that Uranus and Neptune would not grow. This is described in more detail below.

For this set of initial conditions, BF99 found that a significant amount of growth occurred in the first 10^6 years. In particular, they found that 69 mergers had occurred in that length of time (Brunini 2000, pers. comm.). Two large planets formed beyond the orbit of Saturn in a few $\times 10^7$ years. Both these planets were about $9\,M_\oplus$. The inner planet reached this mass in 46 Myrs while the outer planet reached it final mass in 11 Myrs. Their final semi-major axes were at 19.7 AU and 32.6 AU, respectively.

In contrast to BF99, we found only 5 embryo-embryo collisions in the first 10^6 years of our simulation. Between 10^6 and 8×10^6 years, the collision rate was almost an order of magnitude smaller since there were also 5 embryo-embryo collisions during this time [3]. At 8×10^6 years the largest objects in our simulation were only $\sim 0.5\,M_\oplus$, which is only twice the initial embryo mass.

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[3] Again, we tested the sensitivity of this result on the timestep we used and found no relationship.

At this point in the integration, it became clear that we would not produce giant planets in this simulation (or any similar one). Our argument is based on the expected collision rates as calculated by Öpik (1951)-like equations. Our procedure for estimating this is as follows. At particular times during the integration we make a snapshot of the system. For each embryo with semi-major axis between 17 and 27 AU we calculate the probability per year that it will impact any of the other embryos using Öpik (1951)-like equations modified to handle eccentric targets (Bottke et al. 1994). An estimate of the time to the next collision (t_c) is simply the inverse of this rate. For each snapshot, we calculated the mean, minimum, and maximum t_c, which is shown as the black solid curve and the two gray dotted curves in Figure 3A.

Note that at the beginning of the simulation the average t_c is roughly 2.5 Myrs, but can be as small as $670,000$ years. Thus, at this point it may appear possible that giant planets would form. However, the embryos dynamically excite one another on a timescale that is very short compared to the initial t_c. This can be seen in Figure 3B which shows the RMS eccentricity and inclination of the embryos as a function of time. As the eccentricity and inclination increases, t_c also increases until at the time we stopped the integration, it was approaching 2 Gyr. Recall that t_c is the expected time until the next collision. Since it will take many collisions to make a giant planet, it appears that a giant planet will not form in this simulation.

Some insight into the reasons why accretion does not occur can be gained by considering a particle-in-a-box approximation. In the standard way, we can estimate the average time between collisions for an individual embryo as $1/(n\sigma v)$, where n is the volume number density of the embryos, σ is the collisional cross-section of an embryo, and v is the relative velocity of the embryos.

The collisional cross-section includes gravitational focusing and is thus $\pi R^2 f$, where R is the physical radius of the embryo and f is the gravitational focusing factor which is approximately $1 + (v_{esc}/v)^2$, v_{esc} being the escape velocity of the embryo. Although more sophisticated forms of f exist (for example, see Inaba et al. 2001), this form is adequate for our purposes. Note that if we use an RMS inclination of 0.01 in Inaba et al.'s collision rate, we obtain a mean collision time of order $500,000$ years, which is consistent with the minimum collision time we found using the Öpik approximation. Thus our use of the simple form of f is reasonable. We return to this issue below.

The volume number density, n, is simply the number of embryos divided by the volume they occupy. Since the embryos occupy a disk-like region and the radial extent of this disk does not spread significantly during the 8 million years of this simulation, so $n \propto 1/i$. On the other hand, Lissauer & Stewart (1993) show that for a fixed ratio of e to i (which is true here), the relative velocity of an embryo with respect to other embryos is proportional to i. Thus, nv is roughly constant as the system evolves. Changes in t_c are due to changes in the cross-section, σ, which must be due to changes in the gravitational focusing. Indeed, the dotted curve in Figure 3A shows t_c if there were no gravitational focusing (again calculated with the Öpik-like equations). Note that it is roughly constant with respect to time. At the beginning of the integration f is roughly 1000. However, since $f \propto v^{-2}$ for $v < v_{esc}$ (which is true here), $v \propto i$, and the RMS inclination increases from 0.01 at the beginning of the simulation 0.1 at the end (see Figure 3B), f is roughly 10 at the end of the simulation and t_c increased by a factor of 100. So, at the end of the simulation $t_c > 10^9$ years and we do not expect many more mergers to occur.

Given the above result, one may be tempted to argue that Uranus and Neptune could indeed form if initially the disk of embryos were dynamically very cold. This is, if v were initially very small then f, according to the simple formulation, would be very large and t_c would be very short. However, the simple formulation for f breaks down for small v. For example, if we would allow the initial RMS inclination to get as small as 0.001, then the collision time from Inaba et al. could get as low as 50,000 yrs, but cannot get any shorter than that because it saturates at a constant value that is independent of eccentricity and inclination. Figure 3 shows that in our simulation, significant velocity evolution occurs on a timescale that is much shorter than 50,000 years. So, cooling the initial system will not aid in the formation of Uranus and Neptune. Clearly objects the size of Uranus and Neptune will not form by 10^8 years and most likely will not form at all under the physical model studied here.

4. Conclusions

We have investigated two different scenarios for the *in situ* formation of Uranus and Neptune (I89 and BF99). Both of these scenarios hypothesize that Uranus and Neptune formed from several hundred Mars- to Earth-sized objects initially on low-inclination, nearly-circular orbits beyond Saturn. Only the mutual gravitational effects of the embryos and the effects of Jupiter and Saturn were included in this integration.

We find that giant planets do not form in these simulations. Indeed, little accretion occurs at all. The embryos are gravitationally excited by each other and Jupiter and Saturn. Eccentricities larger than 0.25 were achieved in both runs. Such large eccentricities and inclinations significantly decrease the gravitational focusing factor and thus the gravitational cross-section of the embryos. This effectively stops accretion. Thus it seems unlikely that Uranus and Neptune can form from a large number of moderate mass objects.

So, how did Uranus and Neptune form? There are several possible solutions to this problem. First, it may be possible to keep the gravitational cross-section of the embryos large by adding a drag force. Recall that the scenarios studied here assume that 1) Uranus and Neptune formed in a gas-free environment, and 2) the effects of a large number of small objects are not important. Perhaps the inclusion of either gas drag, dynamical friction, or collective gravitational effects (Ward 1988, 1993, Ward & Hahn 1998, Tremaine 1998) that are produced by a gas or particulate disk could keep the orbital eccentricities small enough for the giant planets to form. Initial simulations of this idea generally produce compact planetary systems that must re-expand as the damping force is removed to produce a stable system (Stewart & Levison 1998).

Second, runaway growth of the initial planetary embryos may continue all the way to Uranus/Neptune-sized planets (Bryden et al. 2000). This scenario requires a large supply of low-velocity planetesimals and does not explain why only four giant planets formed in the solar system. A possible mechanism that could help this scenario is the radial migration of the embryos (caused by interactions with the disk) into undepleted regions of disk (Ward & Hahn 1995, Tanaka & Ida 1998, Ida et al. 2000). Third, perhaps Uranus and Neptune formed closer to the Sun (where growth times are shorter) and were subsequently transported outward. One such scenario has already been studied by Thommes et al. (1999). We will continue this line of inquiry in future papers.

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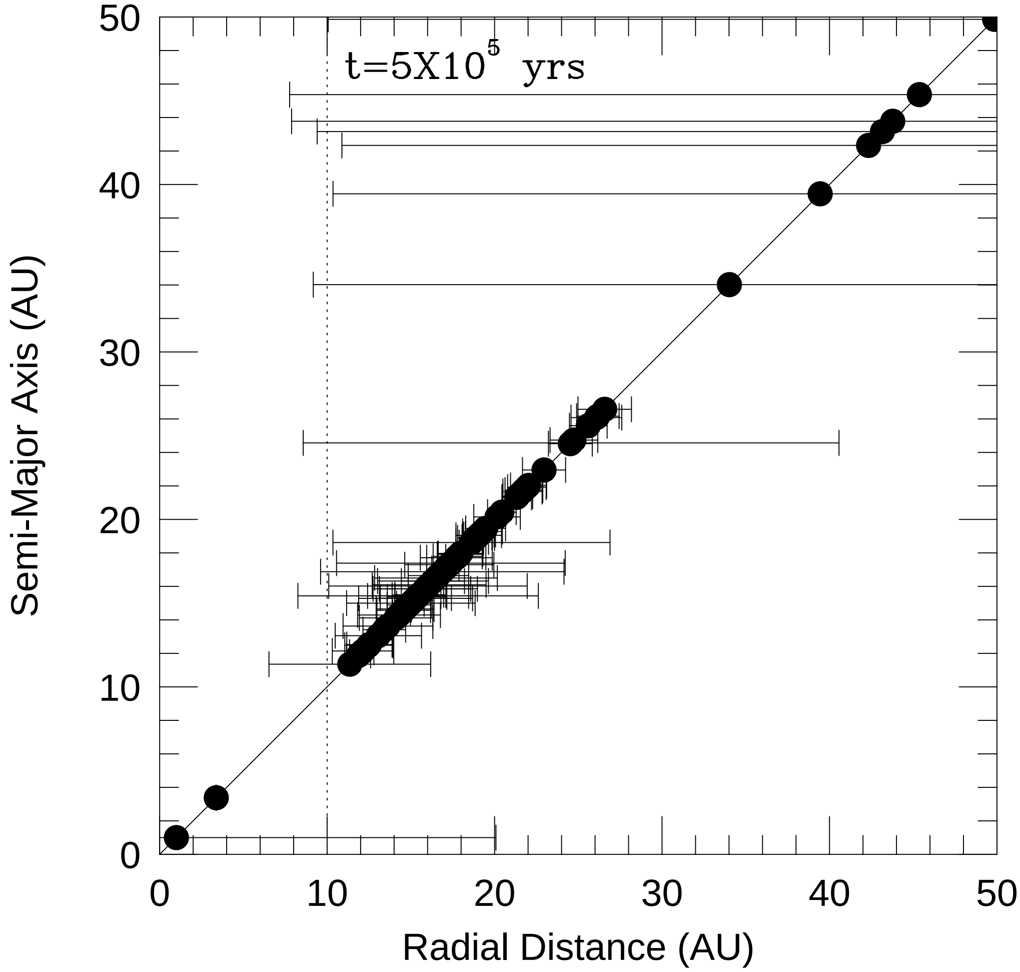
Fig. 1.— The dynamical state of our IP89 run at 5×10^5 years. On the abscissa we plot the heliocentric distance. The data points show the location of the particles' semi-major axes. The horizontal 'errorbars' show the range of radial extent of a particle's heliocentric distance due to its eccentricity. We also plot the particles' semi-major axes on the ordinate. Thus, all the data points are on a diagonal line. The dotted line shows Saturn's semi-major axis.

Fig. 2.— The total amount of mass in the form of embryos at a function of time in our IP89 simulation. At 10^8 years only $32\,M_\oplus$ of material remain which is roughly the sum of the mass of Uranus and Neptune.

Fig. 3.— A) The temporal evolution of our estimate of t_c in BF99 simulation based on a Öpik-like calculation (see text for details). The black solid curve and the gray dotted curves show the mean, minimum, and maximum value of t_c including gravitation focusing. The back dashed curve shows the mean t_c if gravitation focusing is ignored. B) The temporal evolution of the RMS eccentricity and inclination in BF99 simulation.

Figure 1 —



t=5X10^5 yrs
Semi-Major Axis (AU)
Radial Distance (AU)
0
10
20
30
40
50

Figure 2 —

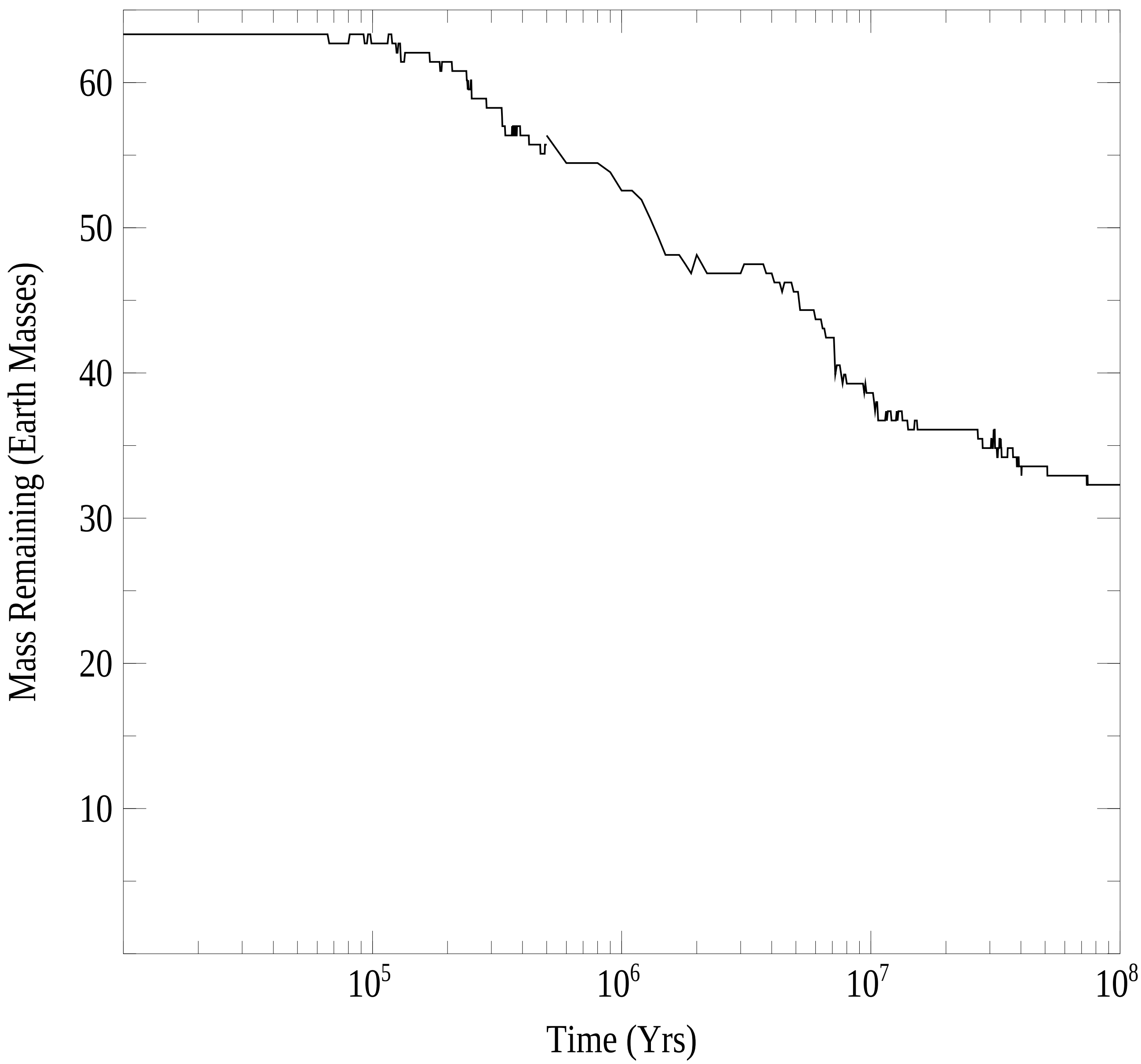
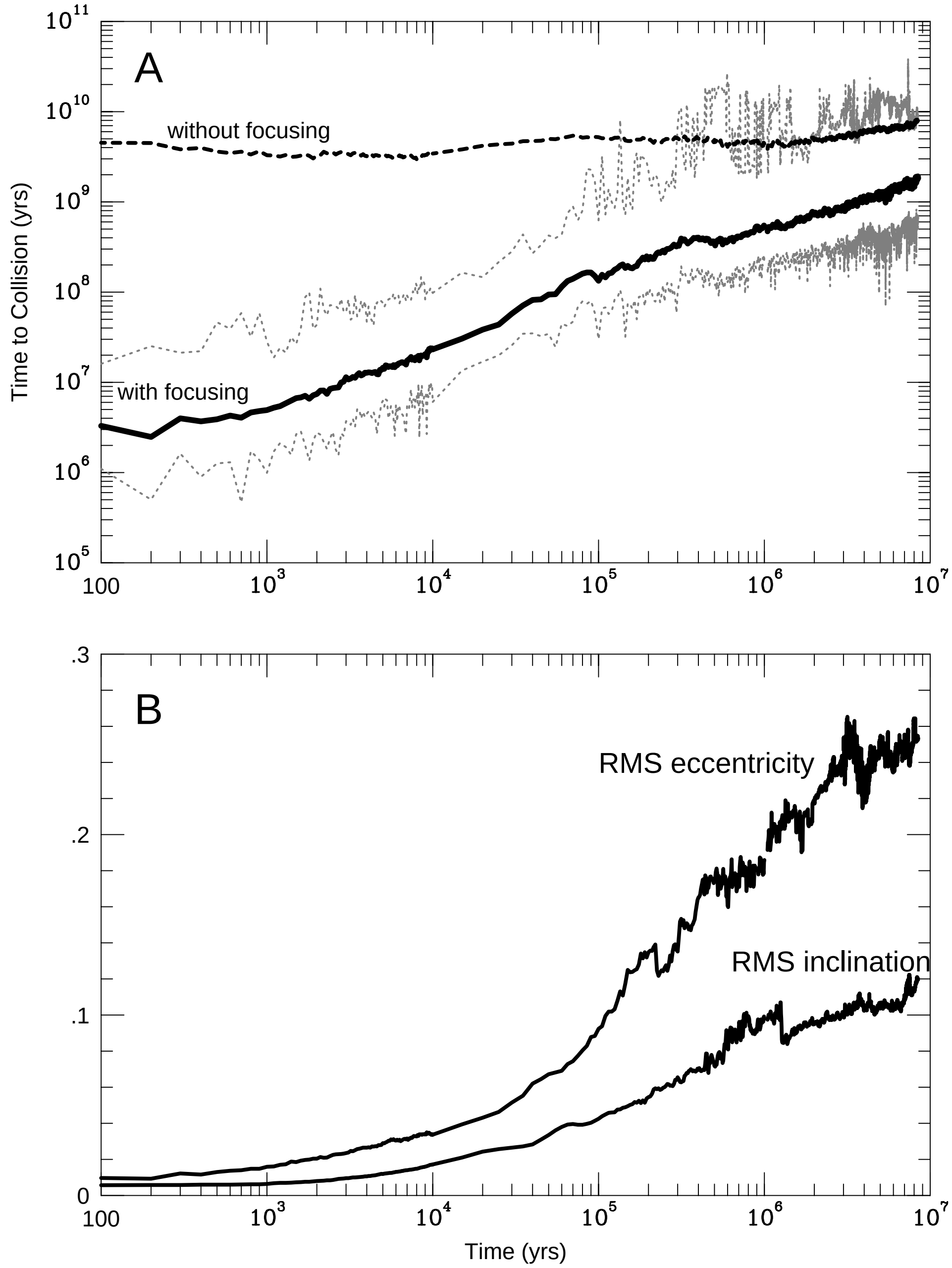

60
50
40
30
20
10
Mass Remaining (Earth Masses)
10^5
10^6
10^7
10^8
Time (Yrs)

Figure 3 —


A
without focusing
with focusing
Time to Collision (yrs)
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
100
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
B
RMS eccentricity
RMS inclination
.3
.2
.1
0
Time (yrs)